1 a $v = 6 + pt + qt^3$

$x = 6t + \frac{1}{2}pt^2 + \frac{1}{4}qt^4 + c$

When $t = 0, x = 0, \quad \therefore \quad c = 0$

When $t = 2, x = 16, \quad \therefore \quad 16 = 12 + 2p + 4q$

$\therefore \quad 4 = 2p + 4q$

$\therefore \quad 2 = p + 2q \quad \cdots (1)$

$a = p + 3qt^2$

When $t = 2, a = 32, \quad \therefore \quad 32 = p + 12q \quad \cdots (2)$

Subtracting (1) from (2) gives

$30 = 10q$

$\therefore \quad q = 3$

Substituting in (1) gives

$2 = p + 2 \times 3$

$\therefore \quad p = -4$

b $v = 6 - 4t + 3t^3$ and $a = -4 + 9t^2$

When $a = 0$, $-4 + 9t^2 = 0$

$\therefore \quad 9t^2 = 4$

$\therefore \quad t^2 = \frac{4}{9}$

$\therefore \quad t = \frac{2}{3}$ (the negative solution is not appropriate)

When $t = \frac{2}{3}$, $v = 6 - 4 \times \frac{2}{3} + 3 \times \left(\frac{2}{3}\right)^3$

$= 6 - \frac{8}{3} + \frac{8}{9}$

$= \frac{38}{9} = 4\frac{2}{9}$

The velocity of the particle is $4\ \frac{2}{9}$ m/s.

2 a Using $s = ut + \frac{1}{2}at^2$ where $s = 0, t = 3, a = -9.8$

$0 = 3u + \frac{1}{2} \times -9.8 \times 3^2$

$\therefore \quad -3u = -44.1$

$\therefore \quad u = 14.7$

The speed projection is 14.7 m/s.

b Consider the last 20 m of the fall, with $u = -14.7, a = -9.8$ and $s = -20$, and use

$v^2 = u^2 + 2as$

$\therefore \quad v^2 = (-14.7)^2 + 2 \times -9.8 \times -20$

$= 608.09$

$\therefore \quad v = \pm\sqrt{608.09}$

and, as the stone is moving in a negative direction,

$v = -\sqrt{608.09}$

≈ -24.66

The speed when the stone hits the ground is 24.66 m/s.

c Using $s = ut + \frac{1}{2}at^2$ and $s = 10, u = 14.7, a = -9.8$

$$10 = 14.7t + \frac{1}{2} \times -9.8t^2$$

$$\therefore \quad 10 = 14.7t - 4.9t^2$$

$$\therefore \quad 4.9t^2 - 14.7t + 10 = 0$$

Using the general quadratic formula

$$t = \frac{14.7 \pm \sqrt{14.7^2 - 4 \times 4.9 \times 10}}{2 \times 4.9}$$

$$\approx 1.043, 1.957$$

The stone is 10 m above the top of the cliff at 1.043 and 1.957 seconds.

d The stone is 5 m above the ground when

$$s = 5 - 20$$
$$= -15$$

Using $s = ut + \frac{1}{2}at^2$ and $s = -15, u = 14.7,$

$a = -9.8$

$$\therefore \quad -15 = 14.7t - 4.9t^2$$

$$\therefore \quad 4.9t^2 - 14.7t - 15 = 0$$

Using the general quadratic formula

$$t = \frac{14.7 \pm \sqrt{14.7^2 - 4 \times 4.9 \times -15}}{2 \times 4.9}$$

$$\approx 3.805 \text{ (negative answer is not appropriate)}$$

The stone is 5 m above the ground after 3.805 seconds.

3 a Consider the first half of the journey where $u = 0, v = V, a = a, s = s_1$

$$v^2 = u^2 + 2as$$

$$\therefore s = \frac{v^2 - u^2}{2a}$$

$$\therefore s_1 = \frac{V^2 - 0^2}{2a}$$

$$= \frac{V^2}{2a}$$

For the second part of the journey, $u = V, v = 0, a = -r, s = s_2$

$$\therefore \quad s_2 = \frac{0^2 - V^2}{-2r}$$

$$= \frac{V^2}{2r}$$

$$\text{Total distance} = s_1 + s_2$$

$$= \frac{V^2}{2a} + \frac{V^2}{2r}$$

$$= \frac{V^2 r + V^2 a}{2ar}$$

$$= \frac{V^2(a + r)}{2ar}$$

The length of the test run must be at least $\frac{V^2(a+r)}{2ar}$.

b i Note that $\frac{2V^2(a+r)}{9ar}$ is less than the value found in part a. So the car does notachieve its top speed for this test run length. Assume it reaches a speedof kV where $0 < k < l$.

Using the same method as part a, $s_1 + s_2 = \frac{k^2V^2(a+r)}{2ar}$

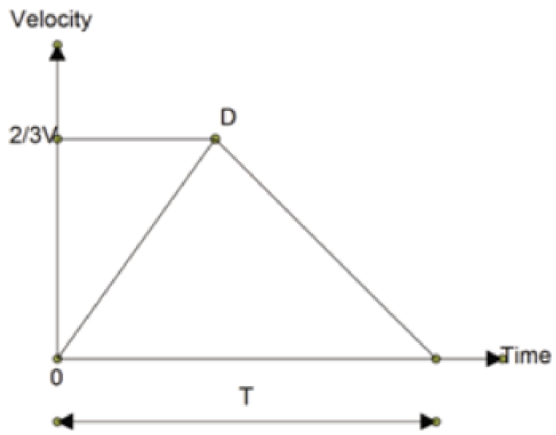
$$s_1 + s_2 = \frac{2V^2(a+r)}{9ar} = \frac{k^2V^2(a+r)}{2ar}$$

Given

$$k^2 = \frac{4}{9}$$
$$k = \frac{2}{3}$$

The car attains a speed of $\frac{2}{3}V$ in this case.

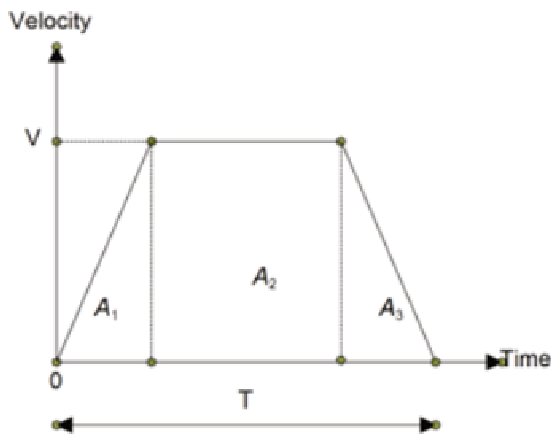
Represent the information on a velocity-time graph.



If T is the total time taken, then

$$\frac{1}{2} \times T \times \left(\frac{2}{3}V\right) = \frac{2V^2(a+r)}{9ar}$$
$$\frac{1}{3}VT = \frac{2V^2(a+r)}{9ar}$$
$$T = \frac{2V(a+r)}{3ar}$$

ii Note that $\frac{2V^2(a+r)}{3ar}$ is greater than the value found in part a. So the car reachesits top speed, stays there for a time, then brakes till it comes to rest.
Represent the information on a velocity-time graph.



From part a, $A_1 + A_3 = \frac{V^2(a+r)}{2ar}$

$$A_2 = \frac{2V^2(a+r)}{3ar} - \frac{V^2(a+r)}{2ar}$$

Hence $= \dfrac{V^2(a+r)}{ar}\left(\dfrac{2}{3}-\dfrac{1}{2}\right)$

$= \dfrac{V^2(a+r)}{6ar}$

If t is the time taken while travelling at constant speed of $\mathbf{V}$, then

$$Vt = A_2$$

$$t = \frac{A_2}{V} = \frac{V(a+r)}{6ar} \qquad (1)$$

Now let T be the total time taken and use the formula for the area of atrapezium (or use two triangles and a rectangle):

$$\frac{1}{2}(T+t)V = \frac{2V^2(a+r)}{3ar}$$

$$T+t = \frac{4V(a+r)}{3ar}$$

$$I = \frac{4V(a+r)}{3ar} - \frac{V(a+r)}{6ar} \text{ (using (1))}$$

$$= \frac{V(a+r)}{ar}\left(\frac{4}{3}-\frac{1}{6}\right)$$

$$= \frac{7V(a+r)}{6ar}$$

iii $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time elapsed}}$

$$= \frac{2V^2(a+r)}{3ar} \div \frac{7V(a+r)}{6ar}$$

$$= \frac{2V^2(a+r)}{3ar} \times \frac{6ar}{7V(a+r)}$$

$$= \frac{4}{7}V$$

4 a Particle X

When X reaches its maximum height $v = 0$.

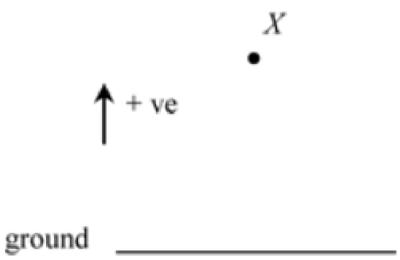
Using $v^2 = u^2 + 2as$

$$0 = 80^2 - 2 \times g \times s$$

$$s = \frac{6400}{2g}$$

$$= \frac{3200}{g}$$

$$\approx \frac{16\,000}{49} \text{ m}$$



When does particle X reach a height 80 m below its maximum height?

This can be found in several ways. Computationally, it is easiest to stay in the same reference system, with up as positive for displacement measured from ground level.

$$\therefore \quad \text{displacement from ground} = \frac{12\,080}{49} \text{ m}$$

Using $\quad s = ut + \frac{1}{2}at^2$

$$\frac{12\,080}{49} = 80t - 0.5 \times 9.8 \times t^2$$

$$\therefore \quad 4.9t^2 - 80t + \frac{12\,080}{49} = 0$$

$$\therefore \quad t = \frac{80 \pm \sqrt{80^2 - 4 \times 49 \times \frac{12\,080}{49}}}{2 \times 4.9}$$

$$= \frac{80 \pm \sqrt{1568}}{9.8}$$

$$= \frac{80 \pm 28\sqrt{2}}{9.8}$$

$$\approx 12.203\ldots \text{ and } 4.122\,65\ldots$$

The larger value gives the time when it goes through the required point on the way down.

Particle X reaches the ground in time given by

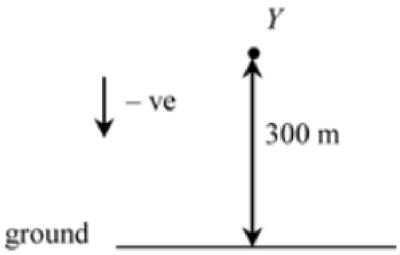
$$0 = 80t - \frac{1}{2} \times g \times t^2$$

$$\text{i.e. } 0 = t\left(80 - \frac{1}{2}gt\right)$$

$$\text{i.e. } t = 0 \text{ or } \frac{160}{g}$$

$$\therefore \quad \text{time taken from } \frac{12\,080}{49} \text{ m to ground} = \frac{160}{g} - 12.203\ldots$$

$$= 4.122\,65\ldots$$



b Particle Y

Using $t = 4.12265\ldots$, $u = v$, $s = 300$ and $a = g$

and $\quad s = ut + \frac{1}{2}at^2$

$$300 = v \times 4.122\,65\ldots + \frac{1}{2} \times 9.8 \times (4.122\,65\ldots)^2$$

$$\therefore \quad v = 52.5677\ldots$$

The value of v is 52.568 m/s, correct to three decimal places.

5 a i Let v m/s be the velocity.

$v = 4 - 10t - 3t^2$

ii Let a m/s^2 be the acceleration.

$a = -10 - 6t$

b i When $t = 0$, $v = 4 - 10(0) - 3(0)^2$

$= 4$

The initial velocity of X is 4 m/s.

ii When $t = 0,\ a = -10 - 6(0)$

$$= -10$$

The initial acceleration of X is $-10\ \mathrm{m/s^2}$.

c For particle Y,

$$a = 2 - 6t$$
$$v = 2t - 3t^2 + c$$

When $t = 0, v = 2$

$$\therefore\ 2 = 2(0) - 3(0)^2 + c$$
$$\therefore\ c = 2$$
$$\therefore\ v = 2t - 3t^2 + 2$$
$$s = t^2 - t^3 + 2t + d$$

When $t = 0, s = 0$

$$\therefore\ d = 0$$
$$\therefore\ s = t^2 - t^3 + 2t$$

When X and Y collide

$$28 + 4t - 5t^2 - t^3 = t^2 - t^3 + 2t$$
$$\therefore\ -6t^2 + 2t + 28 = 0$$
$$\therefore\ 3t^2 - t - 14 = 0$$
$$\therefore\ (3t - 7)(t + 2) = 0$$
$$\therefore\ t = \frac{7}{3} \text{ (negative solution is not appropriate)}$$
$$\therefore\ t = 2\frac{1}{3}$$

X and Y collide after $\frac{7}{3}$ seconds.

d When $t = \frac{7}{3}$, velocity of $X = 4 - 10 \times \frac{7}{3} - 3 \times \left(\frac{7}{3}\right)^2$

$$= 4 - \frac{70}{3} - \frac{147}{9}$$
$$= -\frac{107}{3}\ \mathrm{m/s} = -35\frac{2}{3}\ \mathrm{m/s}$$

velocity of $Y = 2 \times \frac{7}{3} - 3 \times \left(\frac{7}{3}\right)^2 + 2$

$$= \frac{14}{3} - \frac{147}{9} + 2$$
$$= -\frac{29}{3}\ \mathrm{m/s} = -9\frac{2}{3}\ \mathrm{m/s}$$

X and Y are travelling in the same direction and X is travelling faster than Y, catching up and colliding with Y.